ÉRETTSÉGI VIZSGA ● 2014. május 6.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2014. május 6. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In Section II, you are only required to solve four out of the five problems**. When you have finished the examination, write in the square below the number of the problem NOT selected.** *If it is not clear* for the examiner which problem you do not want to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display verbal information. You are also allowed to use any book of four-digit data tables. The use of any other electronic device, or printed or written material is forbidden.

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results are also possible to follow.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to any other theorem(s) will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. Diagrams are also allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem**, indicate clearly** which attempt you wish to be marked.

11. **Do not write anything in the grey rectangles.**

I

1. Solve the following equations on the set of real numbers.

a) $\sin\left(2x-\frac{\pi}{6}\right)=1$

b) $\log_3 x+\log_9 x=6$

a)	5 points	
b)	6 points	
T.:	11 points	

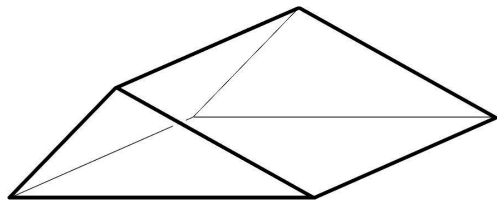
2. **a)** Every edge of a complete graph on 16 points is coloured either red or yellow. After the colouring, there are exactly three red edges from each point. Then two points are selected at random.
What is the probability that the edge connecting the two points selected is red?

b) Starting with another complete graph, a tree graph is obtained by deleting 45 edges.
How many points does this graph have?

(A complete graph is a simple graph in which every pair of points is connected by an edge.)

a)	4 points	
b)	8 points	
T.:	12 points	

3. The Roofers Association is giving each veteran roofer a small solid bronze model of a roof as a souvenir. The base of the bronze object is a square of side 4 cm. The two faces adjacent to the square along two opposite edges are congruent triangles, perpendicular to the base, as shown in the figure. Their lateral edges are 2 cm and 3 cm. The inner dimensions of the rectangular gift box ordered for the bronze model are 4.1 cm × 4.1 cm × 1.5 cm, and the density of the bronze used is $8.2\ \frac{\text{kg}}{\text{dm}^3}$.



Verify by calculation that the bronz model can be put in the gift box, and that its mass does not exceed 100 grams.

T.:	14 points	

4. **a)** Six elements of a set of seven positive integer data are as follows: 10; 2; 5; 2; 4; 2. The seventh element is not known. Given that the mean, mode and median of the data (not necessarily in this order) are three consecutive terms of a strictly increasing arithmetic progression, determine the possible values of the seventh element.

b) How many different four-digit even numbers can be formed out of the digits 0, 1, 2, 3, 4, 5 if all digits need to be different?

a)	9 points	
b)	5 points	
T.:	14 points	

II

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5. In a certain department of a company, the mean age of the male employees is 44 years, and the mean age of the female employees is 40 years. The mean age of all employees of the department is 41.5 years.

a) By what factor is the number of men in the department greater or smaller than the number of women?

In another department, the ratio of the number of men to the number of women was 2:3. Then the company was reorganized, and 7 men and 9 women were transferred from here to some other department. Thus the ratio of the number of men to the number of women remaining changed to 1:2.

b) How many men and how many women stayed in this department?

c) In how many different ways is it possible to create three teams out of 6 women and 3 men, such that each team consist of 2 women and 1 man? (Disregard the order of the teams.)

a)	6 points	
b)	5 points	
c)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

6. **a)** Consider the circle of equation $(x-2)^2+(y+1)^2=45$ centred at O. Let M denote the intersection of the circle with the line e of equation $y = 2$ in the first quadrant. Line e is reflected in the line OM.
Determine the equation of the reflected image of line e.

b) Consider the parabola of equation $y=-x^2+2x+5$ Let P denote the intersection of the parabola with the line $y = 2$ in the first quadrant.
Calculate the slope of the tangent drawn to the parabola at point P.

a)	12 points	
b)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

7. **a)** Use the information below to determine the values of the real parameters a, b and c in the function $f: \mathbf{R} \to \mathbf{R}, f(x) = x^3 + ax^2 + bx + c$.

(1) $f(1) = f(-1) + 4$;

(2) $f'(3) = 10$ (f' is the derivative function of f);

(3) $\int_0^2 f(x)dx = -8$.

b) Show that the polynomial $x^3 - 3x^2 + x - 3$ can be factorised, and hence determine the zeros of the function $g: \mathbf{R} \to \mathbf{R}, g(x) = x^3 - 3x^2 + x - 3$.

a)	11 points	
b)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8. The card game *ulti* is played with a deck of German cards, which consists of 4 suits (hearts, bells, acorns, leaves) with 8 cards in each suit (VII, VIII, IX, X, Under, Over, King, Ace). That is, there are 32 cards altogether.
Dénes, Elemér and Fanni are playing ulti: 10 cards are dealt to each player (at random), and the remaining 2 cards are placed on the table to form the so called talon.

a) Calculate the probability that the two cards placed in the talon after a dealing are of two different suits.

b) Calculate the probability that all the 8 cards of one of the suits are dealt to Elemér.

c) Verify by calculation that the probability of Fanni receiving at least one ace in the deal (rounded to four decimal places) is 0.7966.

d) Determine the (conditional) probability that Fanni received all four aces in the deal, given that she received at least one ace.

a)	4 points	
b)	4 points	
c)	3 points	
d)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. In a certain game, each player started with the same initial score, which may have increased or decreased during the game. Rita and Péter both played well: they kept winning throughout the game, so their scores kept increasing. It was interesting to observe that Rita's score increased by the same factor in each round of the game, and the same was true for Péter's score, too. However, in Péter's case the factor of increase was greater. After the first round, Péter had 20 more points than Rita, after the second round he was leading by 70 points, and by the end of the third round the difference grew to 185 points.
What was the value of the equal initial score?

T.:	16 points	

	number of problem	maximum score	points awarded	maximum score	points awarded
Section I	1.	11		**51**	
	2.	12			
	3.	14			
	4.	14			
Section II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	elért pontszám **egész számra** kerekítve / score rounded to **integer**	programba beírt **egész** pontszám / **integer** score entered in program
I. rész / Section I		
II. rész / Section II		

javító tanár / examiner

jegyző / registrar

dátum / date

dátum / date